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PLANE GEOMETRY

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CONTENTS

APPENDIX

SYMBOLS AND ABBREVIATIONS

+	*plus, or added to*
−	*minus, or diminished by*
=	*equals, or is equivalent to*
≅	*congruent*
≠	*is not equal to*
>	*is greater than*
<	*is less than*
∴	*therefore, or hence*
⊥	*perpendicular, or is perpendicular to*
⊥s	*perpendiculars*
∥	*parallel, or is parallel to*
∥s	*parallels*
~	*is similar to, or similar*
∠	*angle*
∠s	*angles*
△	*triangle*
△s	*triangles*
□	*square*
▱	*parallelogram*
▱s	*parallelograms*
⊙	*circle*
Ⓢ	*circles*
⌒	*arc; as* $\overset{\frown}{AB}$, *arc AB*
adj.	*adjacent*
alt.	*alternate*
ax.	*axiom*
comp.	*complement*
con.	*construction*
cor.	*corollary*
corr.	*corresponding*
def.	*definition*
ex.	*exercise*
ext.	*exterior*
hom.	*homologous*
hy.	*hypotenuse*
hyp.	*hypothesis*
iden.	*identity*
int.	*interior*
isos.	*isosceles*
quad.	*quadrilateral*
rt.	*right*
st.	*straight*
sub.	*substitution*
sup.	*supplementary, or supplement*

NOTE TO PUPIL

Why Study Geometry?

A natural question for you to ask is, "Why should I study geometry?" Here are some reasons:

1. You will learn to appreciate the importance of geometric figures and their properties in the development of modern civilization, including such things as machinery, art, and bridges.

2. You will gain information that will be of invaluable help if you become an engineer, navigator, or pilot; an architect, designer, or decorator; a builder, carpenter, or machinist; an oculist, astronomer, or expert photographer; or if you enter any one of a large number of other occupations.

3. You will be helped to think clearly and logically, no matter what your future occupation may be. A mastery of geometry depends primarily on one's ability to reason things out carefully and correctly. After completing the course, you should be better able to analyze the problems of life and to make intelligent decisions.

The History of Geometry

The ancient Egyptians, as you know, developed a high civilization along the banks of the Nile. Each year, as the river receded from its annual flood, the Egyptians had to redetermine the boundaries of their property. This they did by means of triangles and other geometric figures. They also used geometric facts in building the great pyramids. The Greeks who first learned of these matters from the Egyptians called this kind of knowledge "geometry," which meant "land-measurement."

The Egyptians had been interested in geometry mostly for its practical usefulness. The Greeks loved to solve problems whether they were practical or not, and expanded the geometry they learned from the Egyptians into what is now called demonstrative geometry, the type you will study in this course. The chief practical use made of geometric facts by the Greeks was the creation of beautiful objects and buildings, such as the famous Greek vases and the Parthenon in Athens. Yet, their discoveries proved later on to have many other important practical applications.

Thales (Thay′lees), who lived more than 2500 years ago, was known as one of the Seven Wise Men of Greece and is often called the father of geometry. He was the discoverer of several of the theorems you will study, and is supposed to have been the first man to measure the height of an object by its shadow. Among his own people, he achieved great fame by predicting an eclipse of the sun.

Pythagoras (Pih-thag′oh-rus), another Greek philosopher and mathematician, founded a society of scholars who studied geometry and made several further discoveries. Many other Greek scholars added to the accumulating store of geometric knowledge.

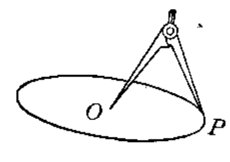
Then, more than 2200 years ago, Euclid (You′clid), a Greek who taught geometry in Egypt, wrote a textbook in which all the geometry known by that time was presented in an orderly and logical arrangement. In all the world's history, there has probably been no other textbook in any subject as widely studied for as long a time. It was translated eventually into most of the important languages of the world, and in various translations was still used in some schools as recently as 1900. For years, studying geometry was called "reading Euclid."

Naturally, all recent textbooks in geometry, including this one, have been strongly influenced by Euclid's text. Truly, multitudes of students have already preceded you down this highway of geometry.

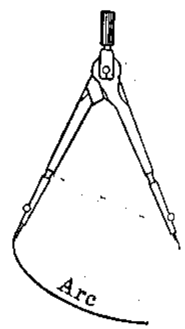
PLANE GEOMETRY

INTUITIVE GEOMETRY — INTRODUCTION

1. The only instruments we shall use in this course are a straightedge (a ruler) and a pair of compasses, except in the Introduction, where we shall use a marked ruler and an instrument called a protractor. (See page 6.)

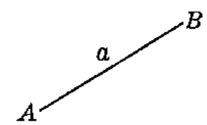


With the ruler we can draw straight lines and with the compasses we can mark off distances, describe arcs, and draw circles.



Construction lines are usually made solid, while any auxiliary lines in a figure are usually dotted. Auxiliary lines are those that it is necessary to draw in order to prove certain facts or relationships about the figure.

A straight line is indefinite in length. Usually we are only interested in a portion of such a line which is called a *segment.* Thus, in the adjoining figure we have the segment called AB or simply a.



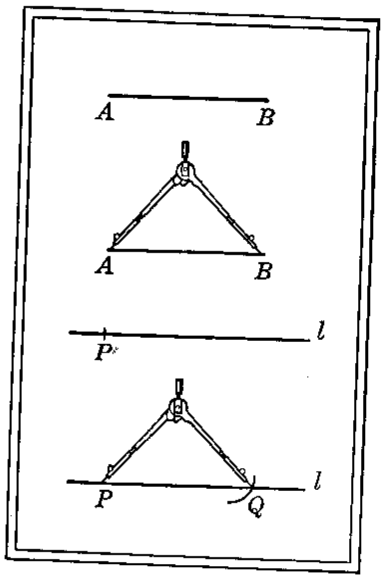
A broken line is one made of segments that differ in direction, as line XY.

EXERCISES

1. Draw two lines meeting in one point.
2. How many lines can you draw through a point?

3. Draw two lines that have two points in common. What can you say about these lines?
4. How many lines can you draw through two points?
5. How many lines can you draw through any two of three points which are not in a straight line?
6. How many lines can you draw through any two of four points no three of which are in a straight line?
7. Can you draw on the blackboard two lines that will not meet no matter how far they are produced in either direction?

2. Problem: *Construct a segment equal to a given segment:*



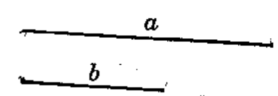
Let the given segment be AB. Set your compasses so that one point is at A and the other is at B. Take

any line l and mark on it a point P. Set one point of your compasses at P and let the other point describe an arc cutting l in Q. Then $PQ = AB$.

EXERCISES

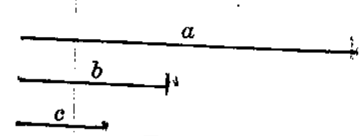
In the following exercises use your compasses in transferring all lengths.

1. Mark off on a line a segment of $\frac{1}{2}''$.
2. Mark off on a line three consecutive segments each equal to $\frac{1}{2}''$.
3. Mark off on a line three consecutive segments of length: $\frac{1}{2}''$, $\frac{3}{4}''$, and $1''$ respectively.
4. Construct a segment twice the length of a given segment.
5. Construct a segment three times the length of a given segment.
6. Construct a segment equal to $a+b$.
7. Construct a segment equal to $a-b$.
8. Construct a segment equal to $2a-b$.



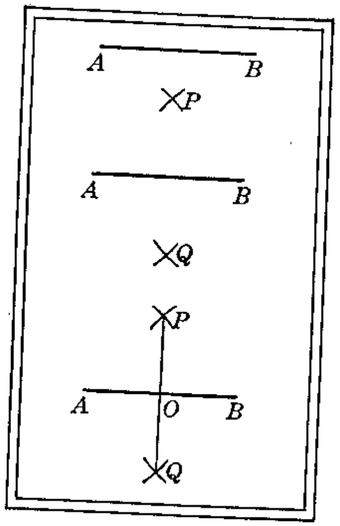
Ex. 6–8

9. Construct a segment equal to $a+b+c$.
10. Construct a segment equal to $a+b-c$.
11. Construct a segment equal to $a-b-c$.



Ex. 9–11

3. Problem: *Bisect a segment.*



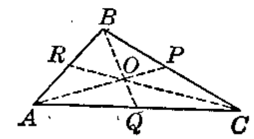
Let the segment be AB. With A and B as centers and with a radius greater in length than half of AB, describe arcs meeting above and below AB, in points P and Q. Draw PQ meeting AB in O. Point O is the desired point.

This can be verified easily by making the construction on thin paper and then folding the paper along PQ and noticing that OB coincides with OA, or by measuring the distances with compasses.

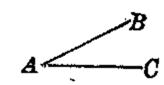
EXERCISES

1. Take a segment 2″ long and bisect it. Measure each segment and see if it is 1″ in length.
2. Divide a segment into four equal parts.
3. Divide a segment into eight equal parts.

4. By the method of § 3 can you divide a segment into three equal parts?
5. By the method of § 3 can you divide a segment into six equal parts? Into any odd number of equal parts?
6. The adjacent figure ABC is called a triangle. The segments AB, AC, BC are called sides and the points A, B, and C vertices. Bisect each side and connect these points with the opposite vertices.

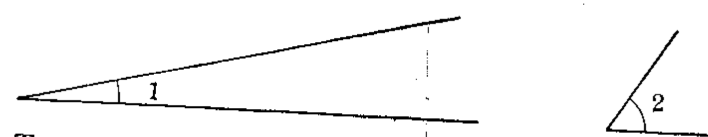


4. Angles. The figure formed by two straight lines drawn from the same point is called an *angle*. In the adjacent figure we have angle A, written $\angle A$, $\angle CAB$, or $\angle BAC$. The point A is called the *vertex*, and the lines AC and AB the *sides*.



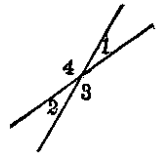
If the line AC is rotated about the point A; from the position AC to the position AB, we say $\angle CAB$ has been generated.

The *size* of $\angle CAB$ depends upon the amount of rotation and not upon the length of its sides. Thus, $\angle 1$ below is smaller than $\angle 2$ even though its sides are much longer.

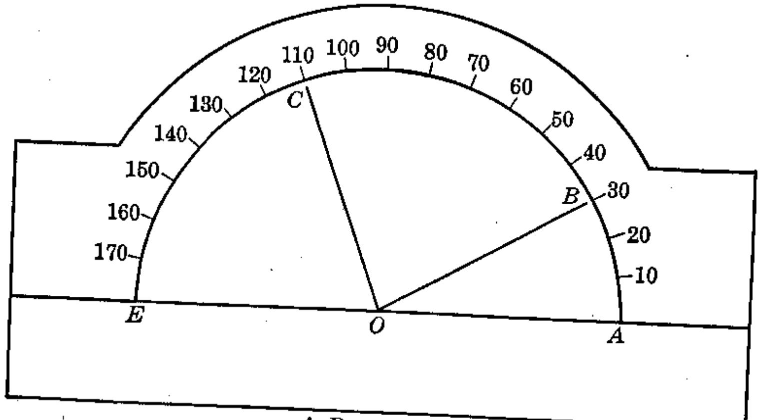


To compare two angles, we place one angle on the other so that their vertices coincide and a side of one angle falls along a side of the second angle. If the other sides coincide, the angles are equal.

5. Vertical angles are two angles with a common vertex and the sides of one prolongations of the sides of the other. Angles 1 and 2 are vertical angles; likewise angles 3 and 4.



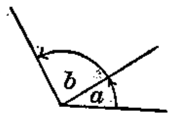
Measurement of angles. The unit of measure of an angle is called a degree. The Babylonians thought there were 360 days in a year so they divided a complete revolution into 360 parts and each part they called a degree (°). The degree is divided into sixty equal parts, each of which is called a *minute* (′), and the minute is divided into sixty equal parts, each of which is called a *second* (″).



A Protractor

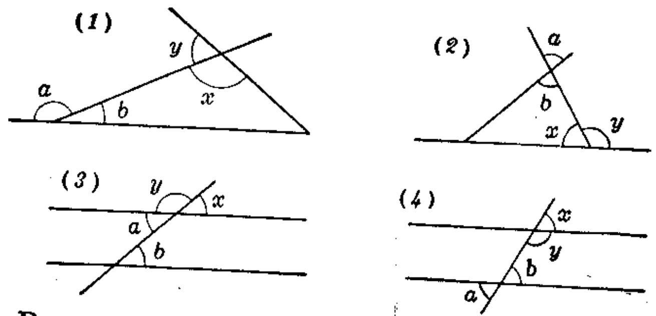
An instrument from Mechanical Drawing, known as a protractor, is very useful in measuring and in drawing angles. If an angle AOC is to be measured, the protractor is set with its center on the vertex O and the base of the protractor on one side of the angle, e.g., OA. It is then noted where OC cuts the protractor scale. $\angle AOC = 110°$; $\angle AOB = ?°$

6. Adjacent angles are two angles having the same vertex and a common side between them.



EXERCISES

In the following figures are $\angle a$ and $\angle b$ adjacent? Are $\angle x$ and $\angle y$ adjacent?

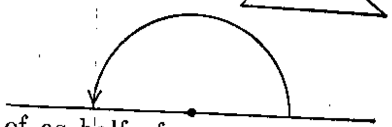


7. Perpendicular lines. Two lines are perpendicular if they meet so as to form two equal adjacent angles. The symbol for perpendicular is $\perp$. Thus, if line a is perpendicular to line b, we write $a \perp b$.

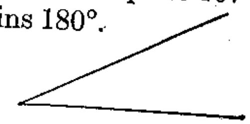
8. Right angle. If the sides of an angle are $\perp$ to each other, the angle is called a right angle. A right angle may also be thought of as a quarter of a complete revolution. Hence, a right angle contains 90°. The symbol for right angle is ∟. Thus, $\angle C$ is a right angle.

C


9. A straight angle is an angle equal to two right angles. A straight angle may also be thought of as half of a complete revolution. Hence, a straight angle contains 180°.

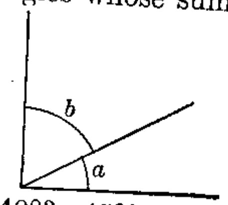


10. An acute angle is an angle less than a right angle.

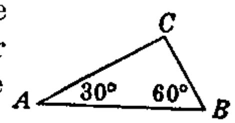


11. An obtuse angle is an angle greater than a right angle but less than a straight angle.

12. Complementary angles are two angles whose sum is a right angle. In the figure at the right, angles a and b are complementary. Each angle is known as the complement of the other. Thus, the complement of 34° is 56° and the complement of 56° is 34°. What is the complement of 30°? 40°? 45°? 60°?



If two complementary angles are adjacent, they form a right angle. But two angles may be complementary and not adjacent. For example, in this figure, $\angle A$ and $\angle B$ are complementary. Why?



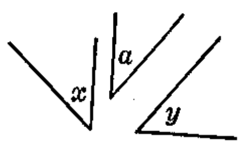
Complements of the same or equal angles are equal.

If $\angle x + \angle a = \text{Rt. } \angle$

And $\angle y + \angle a = \text{Rt. } \angle$

Then $\angle x + \angle a = \angle y + \angle a.$

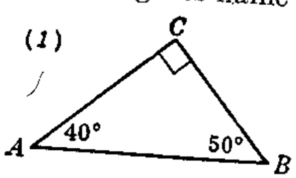
Hence $\angle x = \angle y$ Why?



EXERCISES

In these figures name the complementary angles.

(1)



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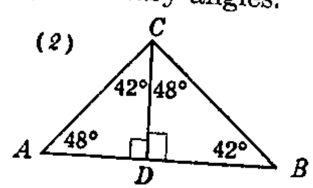
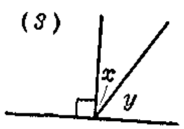


In the following figures tell if $\angle x$ and $\angle y$ are complementary. Give a reason.

(3)



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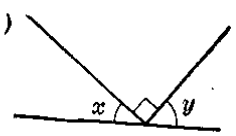

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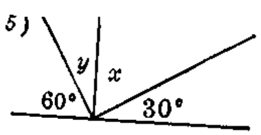
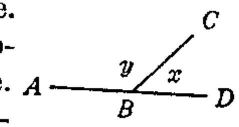
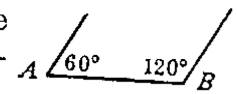


13. Supplementary angles are two angles whose sum is a straight angle or 180°. Thus, angles of 40° and 140° are supplementary. What is the supplement of 60°? 85°? x°?

If two angles are supplementary and adjacent, their exterior sides are in a straight line. Thus $\angle x$ and $\angle y$ are adjacent and supplementary, and ABD is a straight line.



But two angles may be supplementary and not adjacent. Thus in the figure at the right $\angle A$ and $\angle B$ are supplementary. Why?



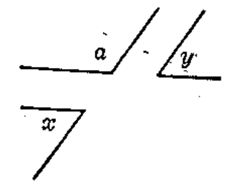
Supplements of the same or equal angles are equal.

If $\angle x + \angle a = 180°$

And $\angle y + \angle a = 180°$

Then $\angle x + \angle a = \angle y + \angle a$

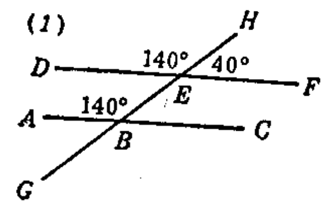
And $\angle x = \angle y$



EXERCISES

In these figures, name the supplementary angles.

(1)



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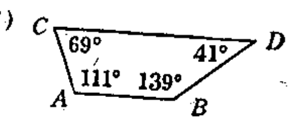

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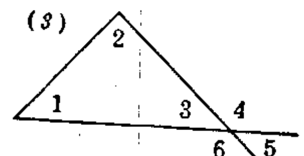
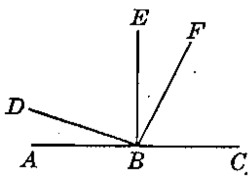


REVIEW EXERCISES ON ANGLES

1. How many degrees are there in a right angle?
2. How many degrees are there in a straight angle?

3. In the adjacent figure $BE \perp AC$. Are the following angles acute, right, obtuse, or straight?

(a) $\angle DBA$. (d) $\angle CBF$.
(b) $\angle EBA$. (e) $\angle CBE$.
(c) $\angle FBA$. (f) $\angle CBA$.



4. How many degrees, minutes, and seconds are there in
(a) $\frac{3}{8}$ of a right angle? (c) $\frac{3}{8}$ of a straight angle?
(b) $\frac{1}{15}$ of a right angle? (d) $\frac{1}{15}$ of a straight angle?

5. If a right angle is subtracted from a straight angle what kind of an angle remains?

6. If an acute angle is subtracted from a straight angle what kind of an angle remains?

7. If an acute angle is added to a right angle what kind of an angle is formed?

8. Is double an acute angle always an obtuse angle?

9. Is half an obtuse angle always an acute angle?

10. If twice an acute angle is an obtuse angle, what number of degrees must the acute angle exceed?

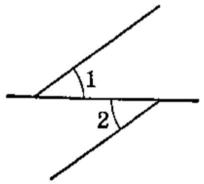
11. What kind of angle is $\frac{2}{3}$ of a straight angle?

12. What kind of angle is $\frac{3}{4}$ of an angle of 175°30′?

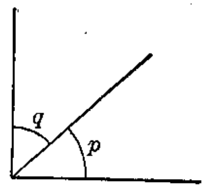
13. Through how many degrees does the minute hand of a watch turn in 15 minutes?

14. What is the angle formed by the hands of a clock at 3 P.M.? 6 A.M.? 4 P.M.?

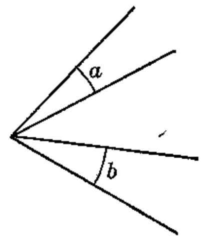
15. Are angles 1 and 2 adjacent? If not, why not?



16. Are angles p and q adjacent? If not, why not?



17. Are angles a and b adjacent? If not, why not?

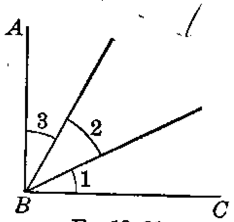


18. In Ex. 16 $\angle p + \angle q = 90°$. Are angles p and q complementary?

19. Given $AB \perp BC$; find $\angle 1$ if $\angle 2 = 27°30'$, $\angle 3 = 42°40'$.

20. Given $AB \perp BC$; find $\angle 2$ if $\angle 1 = 29°40'$, $\angle 3 = 52°20'$.

21. Given $AB \perp BC$; find $\angle 3$ if $\angle 1 = 40°10'11''$, $\angle 2 = 27°13'22''$.



Ex. 19–21

22. What angle is complementary to 47°20′?

23. What angle is supplementary to 47°20′?

24. Among the following angles pick out a pair that are complementary and a pair that are supplementary.

40°, 30°, 10°, 50°, 160°, 45°, 130°.

25. The complement of an angle of $x°$ contains $2x°$. Find the number of degrees in each of the two angles.

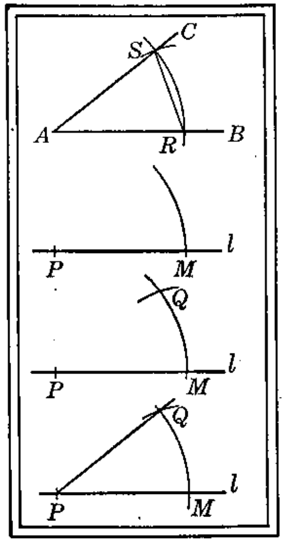
26. The supplement of an angle of $x°$ contains $3x°$. Find the number of degrees in each of the two angles.

27. The greater of two supplementary angles exceeds the smaller by 30°. Find the number of degrees in each angle.

28. If 40° is added to an angle the resulting angle is equal to the supplement of the original angle. Find the original angle.

29. Can you find an angle whose complement and supplement are equal?

14. Problem: *Construct an angle equal to a given angle.*



Let the given angle be BAC. With A as a center and any radius describe an arc cutting AB and AC in R and S. With P, any point on line l as a center and with the same radius, AR or AS, describe an arc cutting l in M. With M as a center and a radius equal to the length RS describe an arc cutting the first arc in Q. Draw PQ. Then $\angle P = \angle A$.

Make the above construction on heavy paper, cut out $\angle P$ and see if it can be made to coincide with $\angle A$.

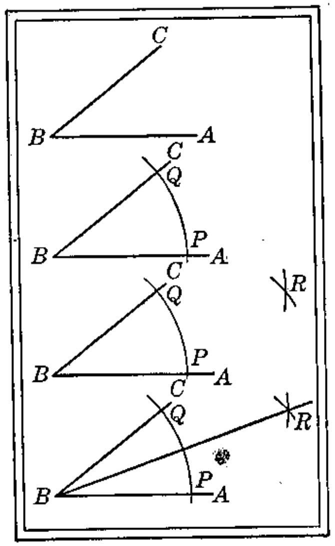
Measure the angles with a protractor. Are they equal?

Remember: In geometry, when one *constructs* a figure, he must use only an unmarked straight edge and a pair of compasses. However, if the instructions are to *draw* a figure, other instruments may also be used, such as a T-square, a protractor, triangles, or marked rulers.

EXERCISES

1. Draw a triangle ABC. (See Ex. 6, § 3.) Construct a triangle $A'B'C'$ having $\angle A' = \angle A$, $\angle B' = \angle B$, but with $A'B'$ twice AB in length. Measure $\angle C'$ and $\angle C$ and see if they are equal.
2. Draw two angles having a common side but different vertices.
3. Draw two angles having the same vertex but with different sides.
4. Draw two acute angles and construct an angle equal to their sum.
5. Draw an obtuse and acute angle. Construct an angle equal to their difference.
6. Draw a triangle. Construct an angle equal to the sum of its angles. What kind of angle is this?

15. Problem: *Bisect an angle.*



Let $\angle ABC$ be the given angle.

With B as a center and with any convenient radius, describe an arc cutting sides BA and BC in points P and Q respectively. With P and Q as centers and with a radius greater than one half the distance PQ describe arcs meeting in a point as R. Draw BR which bisects $\angle ABC$.

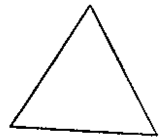
To verify the construction, draw the figure on thin paper; then fold it along BR and see if $\angle RBC$ coincides with $\angle ABR$. Also measure the angles with a protractor.

EXERCISES

1. Draw a triangle and bisect its angles. Do the bisectors appear to pass through a point?

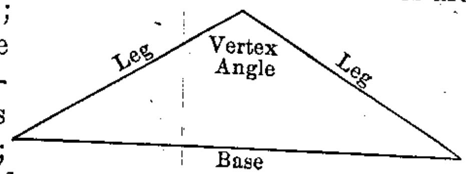
2. Divide an angle into four equal parts.
3. Can the construction of § 15 be used to divide an angle into three equal parts? Into six equal parts?

16. A triangle we have seen is a figure formed by three line segments which meet in three points. (Ex. 6, page 5.) The segments are called *sides* and the points *vertices* of the triangle. A *median* of a triangle is the line drawn from a vertex to the mid-point of the opposite side.

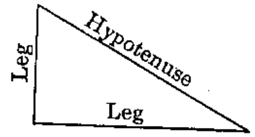


An *equilateral triangle* is a triangle with equal sides. An *equiangular triangle* is a triangle with equal angles.

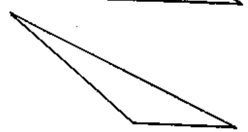
An *isosceles triangle* has two equal sides. The sides are called *legs or arms*; the third side the *base*. The angle opposite the base is called the *vertex angle*; the remaining angles *base angles*.



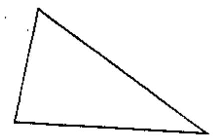
A *right triangle* contains a right angle; the sides making the right angle are called *legs or arms*; and the side opposite the right angle the *hypotenuse*.



An *obtuse triangle* is a triangle containing an obtuse angle.

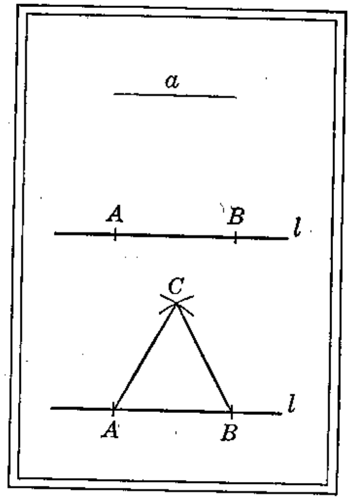


An *acute triangle* is a triangle *all* of whose angles are acute.



17. **Congruent triangles** are those which can be placed one on the other so that they coincide. Line segments, angles, etc., which coincide are called corresponding parts. The symbol for congruence is $\cong$.

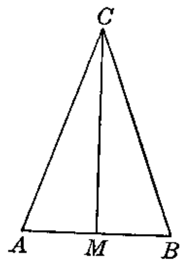
18. Problem: *Construct an equilateral triangle whose sides are equal to a given segment a.*



Lay off on a line l a segment AB equal to the given segment a. With A and B as centers and with a radius equal to a, describe arcs meeting at C. Then the length of each of the segments AC and BC is a. Therefore all of the sides of $\triangle ABC$ are equal and triangle is equilateral.

EXERCISES

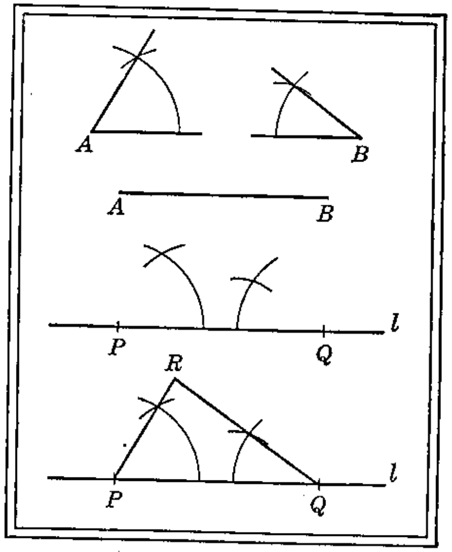
1. Can a right triangle be isosceles?
2. Is every equilateral triangle isosceles?
3. Do you think a right triangle could ever be equilateral?



4. Construct on thin paper an isosceles triangle ABC. Bisect the base, (M), draw CM and fold the triangle along CM and see if $\angle A$ falls on $\angle B$. If it does, what do you think is true about the base angles of an isosceles triangle?
5. Construct an equilateral triangle whose sides are 1″ in length.
6. Construct an equilateral triangle whose sides are 1.5″ in length.
7. Construct an equilateral triangle whose sides are equal to a given segment b.
8. Construct an isosceles triangle whose base is 2″ and whose legs are each 3″.
9. Construct an isosceles triangle with base 3″ and legs 4″.
10. Let a and b be two segments with b greater than a. Construct an isosceles triangle with base a and legs b. Is it possible to construct such a triangle if b is equal to a? If b is less than a? If b is less than $\frac{a}{2}$?
11. Which one of the following statements is true?
    (*a*) Every isosceles triangle is equilateral.
    (*b*) Every equilateral triangle is isosceles.
12. Construct on heavy paper two triangles with sides of length 2, 3, and 4 inches respectively. Cut the triangles out and see if you can place one on the other so as to make them coincide.

13. Construct on heavy paper two triangles whose sides are equal to three given segments a, b, and c, assuming that the sum of the lengths of any two segments is greater than the third. Cut these triangles out and see if you can place one on the other so they will coincide.

19. Problem: *Construct $\triangle ABC$ given the length of AB and angles A and B.*

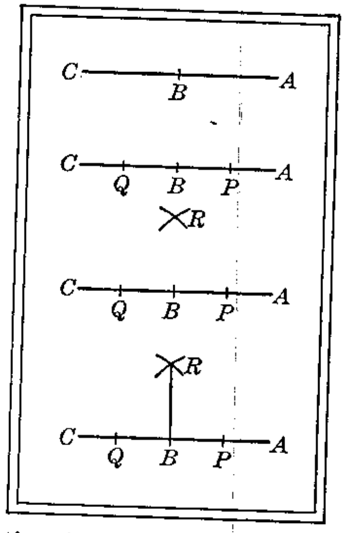


On any line l mark off the segment PQ equal to the segment AB. At P construct an angle equal to $\angle A$ and at Q an angle equal to $\angle B$. Let the sides of these angles meet, as in R. Then $\triangle PQR$ is the required triangle.

EXERCISES

1. On heavy paper construct two triangles ABC and $A'B'C'$ with $AB=A'B'$, $\angle A=\angle A'$, $\angle B=\angle B'$. Cut the triangles out and see if you can place one on the other so they will coincide.
2. Construct $\triangle ABC$ given AB, AC, and $\angle A$.
3. On heavy paper construct two triangles ABC and $A'B'C'$ with $AB=A'B'$, $AC=A'C'$ and $\angle A=\angle A'$. Cut the triangles out and see if you can place one on the other so they will coincide.

20. Problem: *At a point on a given line erect a line perpendicular to the given line.*



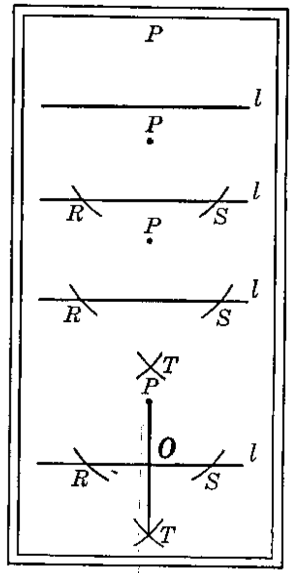
This construction is identical with that of § 15; think of the side BC in § 15 as rotating about B until BC and BA are in a straight line, thus forming the straight angle ABC.

EXERCISES

1. Construct a right triangle whose legs are 1″ and 2″.
2. Construct a right triangle whose legs are 3″ and 4″.
3. Construct a right triangle whose legs are 5″ and 12″.
4. Measure the third side in Exercises 2 and 3 and see if you can find a relation among the three sides of each triangle.
5. Construct a square with side 1″, i.e., a figure with four right angles and four sides of length 1″.
6. Construct an isosceles right triangle with legs 2″ in length.
7. Construct an angle of 45°; 22°30′; 67°30′.
8. Construct an angle of 135°; 33°45′; 11°15′.
9. On a 2″ base, construct two isosceles triangles with legs $1\frac{1}{2}$″ and $2\frac{1}{2}$″ long respectively. Measure and report the sizes of the base and vertex angles in each. What seems to happen to the base and vertex angles of an isosceles triangle when the base remains the same but the length of the sides is increased?
10. On bases of three different lengths, construct three equilateral triangles. Measure and report angle sizes as in Exercise 9. What conclusion would you draw as to the size of angles in equilateral triangles?
11. The earth makes a complete revolution in 24 hours. Through how many degrees does it turn in 2 hours? In 2 hours and 20 minutes?
12. A standard gas pump registers 20 gallons. How many degrees are there between the 2 and 7 gallon marks?

Turn to page 229

21. Problem: *Draw a perpendicular to a line from a point outside the line.*



Suppose the line is l and the point P is not on l. With P as a center and a radius of sufficient length, describe an arc cutting l in R and S. With R and S as centers and with a radius of sufficient length, describe arcs intersecting at T. Draw PT which will be perpendicular to l.

Make the construction on thin paper and fold along line PT and notice that the segments of l coincide.

Measure the adjacent angles SOP and POR with a protractor and see if they are right angles.

EXERCISES

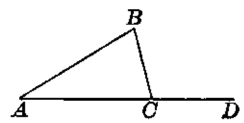
1. Draw a triangle and construct lines from each vertex perpendicular to the opposite side or to that side produced.

2. Construct the adjacent figure in which the opposite sides are equal and all of the angles are right.

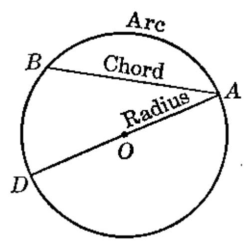
3. Draw $\triangle ABC$ with $AB = BC$. Bisect $\angle B$. Draw the perpendicular from B to AC. Do these appear to be different or identical lines?

4. Draw $\triangle ABC$ with BC twice AB. Bisect $\angle B$. Draw the perpendicular from B to AC. Do these lines appear to be different or identical?

5. Draw any triangle ABC and extend side AC to D. Construct an angle equal to $\angle A + \angle B$ and compare it with $\angle DCB$.

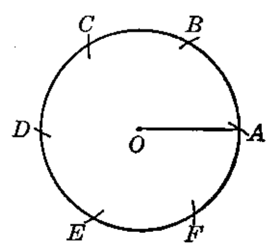


22. The circle is the path traced by a point which moves in a plane so that its distance from a fixed point O is always the same. The point O is called the *center*, the line OA the *radius*, the line AOD the *diameter*, the line AB a *chord*, and the curve AB an *arc*. *All diameters of the same circle are equal and divide the circle into halves.*



A circle passing through the vertices of a triangle is said to *circumscribe* the triangle.

A circle can easily be divided into six equal arcs. Suppose the center of the circle is at O and OA is a radius. With A as a center and OA as a radius, describe an arc cutting the circle at B. Now with B as a center and with



the same radius describe an arc cutting the circle at C. By continuing this process the circle will be divided into six equal arcs.

EXERCISES

1. Draw a circle and three radii. What do you think about the lengths of the radii?

2. Draw a circle and draw a line through its center but terminated by the circle. Such a line is called a *diameter*. Draw three diameters. Are they equal?

3. What do you mean when you say "My radio will receive all stations within a 1000-mile radius"?

4. Draw a circle and a line cutting it in two points.

5. Draw a circle and a line touching it in one point.

6. Draw two circles meeting in two points.

7. Draw two circles with the same radius and touching each other in one point.

8. Draw two circles with the same center but with different radii. In how many points do these circles meet?

9. Draw two circles one inside the other but not touching each other. Is it possible to draw a line touching each circle in only one point? Is it possible to draw a line cutting each circle in two points?

10. Take two points A and B; then see if you can draw at least three circles passing through these points. Do you think you could draw more than three circles that pass through these points?